Subband Coding of Images Using Vector Quantization

PETER H. WESTERINK, DICK E. BOEKEE, JAN BIEMOND, SENIOR MEMBER, IEEE, AND JOHN W. WOODS, SENIOR MEMBER, IEEE

Abstract—Subband coding has proven to be a powerful technique for medium bandwidth source encoding of speech. Recently, some promising results have been reported for extension of this concept to the coding of images. In this paper, a new two-dimensional subband coding technique is presented which is also applied to images. A frequency band decomposition of the image is carried out by means of two-dimensional quadrature mirror filters, which split the image spectrum into 16 equal rate subbands. In the frequency domain, these 16 parallel subband signals are regarded as 16-dimensional vectors and coded as such using vector quantization. In the asymptotic case of high bit rates, a theoretical analysis yields a lower bound to the gain that is attainable by choosing this approach over scalar quantization of each subband with an optimal bit allocation. It is shown that vector quantization in this scheme has several advantages over coding the subbands separately. Experimental results are given and comparisons are made between the new technique presented here and some other coding techniques. This new subband coding scheme has a performance which is comparable to other more complex coding techniques.

I. INTRODUCTION

Since its introduction by Crochiere et al. [1] in 1976, subband coding (SBC) has proven to be a powerful technique for medium rate speech coding. The basic idea of SBC is to split up the frequency band of the signal and then to downsample and code each subband separately using a coder and bitrate closely matched to the statistics of that particular band. Often PCM of DPCM coders are used to code the subbands [1], [2] where the bit rate of each subband coder is determined by a bit allocation which distributes coding errors among the subbands [3]-[5]. By varying this bit assignment a noise spectrum shaping can be achieved which exploits the subjective noise perception of the human ear. This indeed is one of the advantages of subband coding. Recently, other SBC schemes for speech have been presented, in which vector quantization (VQ) is used to code the subbands [6]-[9].

The extension to multidimensional subband filtering was made by Vetterli [10], who considered the problem of splitting a multidimensional signal into subbands, but no coding results were presented in that paper. The first image coding results, together with an approximate theoretical analysis, were presented by Woods and O'Neil [11], [12] who used both DPCM and adaptive DPCM to code the individual subbands. Some initial results on SBC with vector quantization were then presented by Westerink et al. [13]. SBC applied to video conference signals was realized by V. Brandt [14], using temporal DPCM with conditional replenishment.

In this paper, we present a form of SBC that makes use of VQ by exploiting the dependencies between the subbands. In this approach, we form vectors that consist of samples coming from each subband. First, Section II summarizes the extension of subband filters to two dimensions in the case of separable filters. The new subband coding scheme is next presented in Section III. Further, Section IV deals with a mathematical analysis of the coding gain that can be achieved with SBC using VQ. Experimental results are given in Section V for images which are outside the training set, and a comparison is made to other coding techniques. Finally, in Section VI conclusions are drawn.

II. SUBBAND FILTERING

In the subband coding scheme presented in this paper, the image frequency band is split into 16 equally sized subbands, following Woods and O'Neil [11], [12]. This is done hierarchically. First, the signal is partitioned into the four bands shown in Fig. 1, using four separable 2-D digital filters. Each of these four subbands is then demodulated to baseband by a (2 × 2) downsampling. The four resulting signals are then full band at a lower sampling rate. For the 16-band system, this process is repeated by further splitting each subband into four smaller subbands. The resulting 16 subbands are full band at a sampling rate which is a factor four smaller than the original in each dimension.

After encoding, transmission and decoding the image must be reconstructed from the decoded subbands. For that purpose the subbands are upsampled by a factor (2 × 2) and suitably bandpass filtered to eliminate the aliased copies of the signal spectrum which result due to downsampling. The original signal is then reconstructed by adding each of the four upsampled and filtered subbands. For the 16-band system the process is repeated in the tree-like fashion as shown in Fig. 2.

When the ideal filter characteristics of Fig. 1 are approximated with FIR filters, the downsampling in the splitting stage will cause aliasing errors that are not removed during

Fig. 1. Initial four-band partitioning of the image frequency spectrum.
reconstruction. Both in speech and in images this effect is found to be unacceptable [3], [15] and needs to be removed. For that purpose the quadrature mirror filter (QMF) technique was introduced in 1-D subband filtering by Esteban and Galand [16] and was later extended to the multidimensional case by Vetterli [10]. Vetterli also shows that as a special case of the general m-D technique it is possible to consider separable quadrature mirror filters that reduce the filter problem again to one dimension.

In the case of 1-D subband filtering, a 1-D filter pair $h_0(n)$ and $h_1(n)$ is chosen for splitting a signal into two subbands. Then their corresponding transfer functions are $H_0(\omega)$ and $H_1(\omega)$ which are low-pass and high-pass, respectively. The QMF approach now consists of defining the 1-D reconstruction filters $F_0(\omega)$ and $F_1(\omega)$ according to [16]

$$F_k(\omega) = 2(-1)^k H_k(\omega) \quad k = 0, 1.$$  

(1)

By choosing the 1-D filters this way it follows that for perfect reconstruction the QMF pair $h_0(n)$ and $h_1(n)$ must satisfy [16]

$$h_0(n) = h_0(L-1-n) \quad 0 \leq n \leq L/2 - 1,$$  

(2a)

$$h_1(n) = (-1)^n h_0(n)$$  

(2b)

$$\|H_0(\omega)\| + \|H_1(\omega)\| = 1.$$  

(2c)

Unfortunately, the filter requirement in (2c) cannot be exactly met for filter lengths other than $L = 2$ or $L$ approaching infinity. However, it can be very closely approximated for modest values of $L$ and can be obtained with the aid of an optimization procedure [17].

Perfect reconstruction is possible by leaving the QMF approach. By dropping the coefficient symmetry condition of (2a) and by choosing reconstruction filters $F_k(\omega)$ that are different from those in (1), Smith and Barnwell [18] showed that it is possible to design filters for perfect reconstruction of a 1-D input signal. Actual filter coefficients are presented in a more recent paper [19]. By analogy with Vetterli’s construction [10], Smith-Barnwell filters can be used to design 2-D separable filters which will result in perfect reconstruction of a 2-D input signal. Galand and Nussbaumer [20] proposed an extension of the original method from [18], allowing a slight overlap ripple in the reconstructed signal and present filter coefficients for 16- and 20-tap filters.

In this paper, we follow the line of work by Woods and O’Neil [11], [12] and also allow a small ripple in the reconstructed signal (\( \leq 0.025 \text{ dB} \)) by using 2-D separable QMF’s. For the four band partitioning as shown in Fig. 1 the

2-D splitting filters can be written as

$$H_f(\omega_1, \omega_2) = H_f(\omega_1)H_f(\omega_2) \quad 0 \leq i, j \leq 1$$

(3)

where $H_f(\omega)$ and $H_f(\omega)$ are a 1-D QMF pair. The 2-D reconstruction filters are also separable and are obtained using (1) and (3), yielding

$$F_f(\omega_1, \omega_2) = 4(-1)^{i+j} H_f(\omega_1, \omega_2) \quad 0 \leq i, j \leq 1.$$  

(4)

In our coding simulations, we used the filter coefficients of the 1-D 32-tap QMF designated as 32D in [17].

III. SUBBAND CODING SCHEME

Subband coding of speech was partially motivated by the idea that the individual subbands could be coded more efficiently than the full band signals thus yielding an overall bit rate reduction with the same amount of distortion. The subbands are coded separately where the bit rate per subband has to be determined by some sort of bit allocation procedure, optimizing a chosen error criterion. In the most simple form, each subband is coded using a scalar quantizer (SQ) matched to the statistics of that band with a certain preassigned bit rate. More complex coding systems for speech may incorporate adaptive PCM [1], DPCM, or adaptive DPCM [2]. Both DPCM and adaptive DPCM have also been applied to images [11], [12]. All of these subband coding schemes make use of only the within-band dependencies and a variable bit allocation possibility between the bands.

The output of the QMF bank consists of 16 signals all of which are sampled at the same rate. It is therefore natural to consider corresponding samples as vectors in a 16-dimensional space and to then encode this vector source as such using a vector quantizer. The advantages of designing the system in this fashion are several. There is no bit allocation procedure needed, while noise shaping can still be achieved by choosing a suitable distortion measure for the VQ. Furthermore, both the linear dependencies (correlations) and the nonlinear dependencies (being all other statistical dependencies) between the subbands are exploited, and as is well known, a VQ has a better sphere-packing capability than an SQ, which partitions space into multidimensional rectangular blocks. These properties can directly be derived from the various processes at work in a VQ as described by Makhoul et al. [21].

It is also possible to incorporate the same predictive and adaptive techniques as are used in SQ schemes in this vector-based subband coding concept, but in this paper, we will only consider the described elementary vector-SBC system to minimize coder complexity. The total subband coder system as outlined above is shown schematically in Fig. 3.

IV. ASYMPTOTIC CODING GAIN

To show the importance of taking this approach, in this section we will calculate the coding gain that is obtained when applying VQ to the subband signals as described in the previous section, instead of taking a set of scalar quantizers to
quantize each subband separately. The coding gain \( G_{\text{VQ}} \) is defined as the ratio between the distortion in the case of scalar quantizers for each subband to the distortion in the case of a single vector quantizer. This quantity can be expressed as

\[
G_{\text{VQ}} = \frac{D_{\text{SQ}}(R)}{D_{\text{VQ}}(R)}
\]

(5)

where \( k \) is the vector dimension (in our case the number of subbands) and \( R \) is the total bit rate in bits per vector. In the case of SQ, the distortion of each subband is computed for an optimal bit assignment to each quantizer.

In 1966, Algazi [22] derived an expression for the distortion-rate function for a probability density function (pdf) optimized scalar quantizer, denoted here by \( D_{\text{SQ}}(R) \) (where \( R \) is in bits per sample because the vector dimension is one). However, in that paper he assumed a small distortion, or equivalently, a large bit rate \( R \). The resulting SQ performance is therefore called asymptotic and is a function of the pdf \( p(x) \) and the \( \ell^r \) power difference distortion measure \( d \), which is defined as

\[
D(x) = |x - q(x)|^r, \quad r \geq 1
\]

where \( q(x) \) is the quantization of \( x \). Then the asymptotic distortion-rate function is given by [22]

\[
D_{\text{SQ}}^r(R) = \frac{2^{-r}}{r+1} \int \left[ \frac{1}{(r+1)} \right]^{(r+1)} dx.
\]

(7)

Experimental results [3] for some special cases for \( p(x) \) point out that the approximation of (7) is accurate to within a few percent for values of \( R = 7 \) bits per sample or larger. Unfortunately, no useful expressions have been found that are also applicable for lower bit rates. A similar result can be derived for VQ and therefore in the following only the asymptotic case of large values for \( R \) is considered.

Taking (7) as a starting point, the problem of finding the optimal bit assignment \( R_i \) for the scalar quantizer of subband \( i \) is posed as minimizing the total resulting distortion \( D_{\text{SQ}}(R) \) with a certain total bit rate \( R \). The total distortion is here defined as the mean of the subband distortions. Noise shaping, which would imply weighting of the distortions, is not considered because it has no real contribution to the actual problem. The problem can thus be formulated as

\[
\text{minimize } D_{\text{SQ}}^k(R) = \frac{1}{k} \sum_{i=1}^{k} D_{\text{SQ}}^r(R_i)
\]

subject to \( R = \sum_{i=1}^{k} R_i \).

(8a)

(8b)

The solution to this constrained minimization problem as derived in the Appendix is

\[
D_{\text{SQ}}^r(R) = \frac{2^{-r}}{r+1} \left[ \int \left[ \frac{1}{(r+1)} \right]^{(r+1)} dx \right]^{(r+1)/k}
\]

(9)

which is a function of \( r, k, R \) and of the pdf’s \( p_i(x_i) \) of the subbands.

As mentioned above a similar asymptotic approximation as in the scalar case is known for a vector quantizer. Zador [23] gives an expression for the distortion-rate function of a VQ in the asymptotic case where the bit rate \( R \) is high. The \( k \)-dimensional \( r \)th power difference distortion measure that is used is defined as

\[
D^k = \frac{1}{k} \sum_{i=1}^{k} |x_i - q(x_i)|^r
\]

(10)

and is consistent with the distortion measure used in the scalar case, which follows from (6) and (8a). The distortion-rate function is given by

\[
D_{\text{VQ}}^r(R) = A(k, r) 2^{-r/(r+k)} \left[ \int \left[ \frac{1}{(r+k)} \right]^{(r+k)} dx \right]^{(r+k)/k}
\]

(11)

The constant \( A(k, r) \) is a function of the vector dimension \( k \) and of \( r \) and represents how well cells can be packed in \( k \)-dimensional space. However, the problem is that \( A(k, r) \) is known explicitly only for a very few cases. For values of \( k \) other than \( k = 1 \), only \( A(2, 2) \) is known exactly. Fortunately, useful upper and lower bounds are available for \( A(k, r) \) that are fairly tight [24]. The density function \( p(x) \) is the \( k \)-dimensional joint pdf of the vector process \( x \). Unfortunately, very little is known of multidimensional pdf’s and the possibility to measure them. Therefore, to be able to compare (11) to the scalar case of (9) for some specific pdf’s, the vector elements (the subbands) are here assumed to be independent, yielding a pessimistic approximation of \( D_{\text{VQ}}(R) \). Then the joint pdf \( p(x) \) is separable and can be written as

\[
p(x) = \prod_{i=1}^{k} p_i(x_i)
\]

(12)

and the VQ performance of (11) will simplify to

\[
D_{\text{VQ}}^r(R) = A(k, r) 2^{-r/(r+k)} \left[ \int \left[ \frac{1}{(r+k)} \right]^{(r+k)} dx \right]^{(r+k)/k}
\]

(13)

As a result, the gain as defined in (5) can be expressed as

\[
G_{\text{VQ}} = \frac{2^{-r}}{(r+1)A(k, r) \prod_{i=1}^{k} \left[ \int \left[ \frac{1}{(r+k)} \right]^{(r+k)} dx_i \right]^{(r+k)/k}}
\]

(14)

Note that \( G_{\text{VQ}} \) does not depend on the bit rate \( R \). Again, it must be stated here that this result applies only when the number of bits assigned per subband \( R_i \) is greater than zero and, in fact, large.

For three special cases, \( G_{\text{VQ}} \) has been evaluated and the results are shown in Table I. It can be seen that in all three cases, the gain \( G_{\text{VQ}} \) is independent of the variances of the pdf’s. This implies that a VQ implicitly establishes an optimal distribution of bits between the subbands.

In Fig. 4, the curves of \( G_{\text{VQ}} \) are plotted as a function of the vector dimension \( k \) for the three cases considered in Table I. As a distortion measure the mean squared error \( (r = 2) \) is used. For \( A(k, 2) \) the upper bounds from [24] are taken, again to get an indication of the minimum gain \( G_{\text{VQ}} \) that is attainable. These values for \( A(k, 2) \) are also responsible for the nonsmoothness of the curves. As Fig. 4 shows, even in the case of independent vector elements VQ, has a gain over SQ.

Measurements point out that the distribution of the image subbands can be approximated quite well with a Laplacian pdf which in our case of 16 subbands yields a minimum gain of
TABLE I
ASYMPTOTIC LOWER BOUND TO THE GAIN OF VQ OVER SQ OF EACH SUBBAND FOR THREE PDF'S

<table>
<thead>
<tr>
<th>PDF</th>
<th>p(x) (ZERO MEAN)</th>
<th>GAIN G_vq</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAUSSIAN</td>
<td>( \frac{1}{\nu} \exp \left( -\frac{x^2}{2\nu^2} \right) )</td>
<td>( 2^{-\nu} \left( r^{(1)^{r^{1/2}}/2 \left( A(k, r) \right)^{(r/2)} \left( 2k \right)^{r/2}} \right) )</td>
</tr>
<tr>
<td>LAPLACIAN</td>
<td>( \frac{1}{\nu \sqrt{2}} \exp \left( -\frac{</td>
<td>x</td>
</tr>
<tr>
<td>UNIFORM</td>
<td>( \frac{1}{\nu \sqrt{2}} \left( \frac{1}{\nu \sqrt{2}} \right)^{s} \times \nu \sqrt{2} )</td>
<td>( 2^{-\nu} \left( r^{(1)^{r^{1/2}}/2 \left( A(k, r) \right)^{(r/2)} \left( 2k \right)^{r/2}} \right) )</td>
</tr>
</tbody>
</table>

![Fig. 4](image-url)  
Asymptotic gain G_vq of VQ over SQ as a function of the vector dimension k: (a) Gaussian pdf, (b) Laplacian pdf, (c) uniform pdf.

Almost 4. It can reasonably be expected that the asymptotic gain will be higher because the subbands are definitely not independent and because upper bounds on \( A(k, 2) \) are used to calculate G_vq.

V. SIMULATIONS AND RESULTS
Coding simulations were carried out using the subband coding system of Fig. 3. The quadrature mirror filter banks for splitting the image into 16 subbands and for reconstructing the image were implemented using the 2-D separable filters as described in Section II. These 2-D QMF’s were realized as a circular convolution by means of the 2-D fast Fourier transform (FFT). The 1-D QMF that is used to construct the 2-D QMF’s is the 32-tap filter designated as 32D in [16]. This filter has a transition bandwidth of 0.043 radians and an overall passband ripple of 0.025 dB. The stopband rejection varies from 38 to 48 dB.

The vector quantizer is a full search vector quantizer based on the well-known mean squared error (MSE) distortion measure; no noise shaping is applied. The codebooks that are searched by the VQ have been generated using the LBG algorithm, due to Linde, Buzo, and Gray [25]. The algorithm uses a training set and an initial guess of the codebook to arrive iteratively at a (locally) optimal codebook. The initial guess for generating a codebook of rate \( R \) is obtained using the “splitting” technique [25], in which a codebook of rate \( R - 1 \) is split into a double-sized codebook of rate \( R \). Splitting is done by the addition and subtraction of a splitting vector to each vector in the codebook. By taking the centroid of the entire training set as the codebook of rate \( R = 0 \), all codebooks up to a certain desired rate can be generated by repetitive use of splitting followed by the LBG algorithm.

For the coding simulations, all codebooks were generated for rates \( R = 0 \) up to \( R = 12 \), using a training set consisting of five different images. All images used for the experiments are of size 256 \times 256 pixels and have 8-bit gray levels. Fig. 5 shows the coding results for the “lady with hat” image (“LENA”). LENA was not included within the set of five images which was used to generate the codebooks so that Fig. 5 shows results of coding outside the training set. Fig. 5(a) is the original 256 \times 256 image with 8 bits per pixel. Fig. 5(b) shows the result of coding at 0.50 bits per pixel for which a codebook of rate \( R = 8 \) was used. Using a codebook of rate \( R = 10 \) yields the coding result as shown in Fig. 5(c) which is at 0.63 bits per pixel. Clearly, both in Fig. 5(b) and (c), the coding degradations can be seen in the vicinity of edges and in high-frequency areas (such as the feather). These coding errors, however, appear not to be annoying to the human observer, being a very advantageous property of subband coding.
Fig. 6. SBC with VQ results on the "man's face" image: (a) original, (b) 0.50 bits per pixel, and (c) 0.63 bits per pixel.

To evaluate the coder performance numerically, the signal-to-noise ratio (SNR) between the original image \(x(m, n)\) and the processed image \(\hat{x}(m, n)\) has been calculated where the SNR is defined as

\[
\text{SNR} = 10 \log_{10} \frac{255^2}{E[(x(m, n) - \hat{x}(m, n))^2]}.
\]

Fig. 7 shows the SNR values of the coding simulation results for the image LENA where all codebooks up to a rate of \(R = 12\) are used. To compare the coder performance, the performances of some other image coding techniques applied to LENA are also shown in Fig. 7. Considered were the following four methods: subband coding using adaptive DPCM (c) [11], [12], spatial VQ (d) and differential VQ (e) both inside a single image training set consisting of LENA [26], and an adaptive discrete cosine transform (DCT) coding technique (f) [27]. All dashed line plots are taken from [15] and [26]. As can be seen the coder performance of our SBC using VQ is comparable to these other coding techniques in the bit rate region between 0.50 and 0.70 bits per pixel. The SBC using adaptive DPCM, however, still outperforms all techniques but has the highest complexity.

In Fig. 8 the SNR values for coding LENA at 0.63 bits per pixel are shown as a function of the number of training images. For coding inside the training set, curve a) shows a decreasing performance at increasing training set size. This is not surprising, since the codebooks then become more general. Curve b), which is for coding outside the training set, however, shows that adding images to the training set of two
images does not increase the SNR values anymore. Apparently, for LENA all training relevant statistical information is also contained in just these first two images of the training set. The additional three images do have their use for training, as similar experiments on other images show an increasing performance at increasing training set size up to all five training images.

Finally, in Fig. 9 experimental results are shown on the coding gain $G_{SQ}$ of SBC using SQ over SBC using SQ. As is to be expected the coding gain starts off at 1.0 for $R = 0$ bits per vector, since all subbands are then coded with just their mean value (being zero) by both coding methods. As the bit rate becomes larger the coding gain increases, no doubt towards an asymptotic value for high bit rates. This final value will depend on the type of image, the training set, and the set of quantizers used. The estimated value in Section IV of 4.0 in this case seems to be too optimistic for the low-bit rates subband coders are working on.

VI. CONCLUSIONS

In this paper, we have described a new subband coding scheme for images which has several advantageous properties. These follow directly from the way SBC and VQ are incorporated into the system. First, by splitting the image into subbands it is possible to include noise shaping between the subbands. However, in contrast to coding each subband separately as is usually employed, no bit allocation procedure is necessary, while noise shaping is performed by the vector quantizer once an appropriate distortion measure has been chosen.

In general, SBC has good subjective properties. Blocking effects which are quite annoying to a human observer may appear when an image is coded by using spatial VQ or by using block transform coding, such as DCT-coding. This type of distortion does not occur in the new SBC technique because the vector quantizer is designed across the subbands.

The complexity of the filtering part in Fig. 3 is comparable to transform coders when pseudo-QMF's are used for splitting and reconstruction [10]. Coder complexity, however, is relatively low, at least when the codebook that has to be searched is not too large ($R \leq 12$). Although a vector quantizer consumes much CPU time in coding simulations (especially in training), VQ is a suitable technique for hardware implementation and can therefore be very well used as a basis for more complex subband coders that incorporate predictive and adaptive techniques as well as vector quantization. An example of such an extension can be found in [28].

The choice of vectors as described in this paper is one of many possible choices. Another form of doing VQ might be by partitioning each subband into blocks and to code the subbands separately using a vector quantizer. Unfortunately, this approach will increase the coder complexity enormously. These, and other subband coding techniques are presently subject of investigation, both experimentally and theoretically.

APPENDIX

The problem of finding an optimal bit assignment for each subband scalar quantizer is formulated in Section IV as

\[
\begin{align*}
\text{minimize} & \quad D_{SQ}(R) = \frac{1}{k} \sum_{i=1}^{k} D_{SQ}(R_i) \\
\text{subject to} & \quad R = \sum_{i=1}^{k} R_i,
\end{align*}
\]

This minimization problem can be solved by using Lagrange multipliers. The equation to solve using this technique is

\[
\frac{\partial}{\partial R_i} \left[ \frac{1}{k} \sum_{i=1}^{k} 2^{-r} \frac{r}{2} - \gamma R_i \alpha_i(p, r) - \lambda \left( R - \sum_{i=1}^{k} R_i \right) \right] = 0,
\]

where $\lambda$ is a Lagrange multiplier. Setting, for convenience,

\[
\alpha_i(p, r) = \left( 1 + (p / (x_1))^{1/(r+1)} \right)^{(r+1)}, \quad (r \geq 1)
\]

and using (7) then (17) can be written as

\[
\frac{\partial}{\partial R_i} \left[ \frac{1}{k} \sum_{i=1}^{k} 2^{-r} \frac{r}{2} - \gamma R_i \alpha_i(p, r) + \lambda \left( R - \sum_{i=1}^{k} R_i \right) \right] = 0.
\]

Taking the partial derivative with respect to $R_i$ yields

\[
\frac{1}{k} \sum_{i=1}^{k} 2^{-r} \frac{r}{2} \ln 2 \alpha_i(p, r) + \lambda = 0,
\]

which, after rewriting, gives an expression for $R_i$ in terms of $\lambda$

\[
R_i = \frac{1}{r} \log_2 \left( \frac{\alpha_i(p, r)(r \ln 2)^{2^{-r}}}{\lambda k(r+1)} \right).
\]

The Lagrange multiplier is next calculated by substituting (21) into the constraint of the minimization problem, (16b), yielding

\[
\lambda = \frac{r \ln 2}{k(r+1)} 2^{-r} (\ln 2)^{2^{-r}} \left[ \sum_{i=1}^{k} \alpha_i(p, r) \right]^{1/k}.
\]

Substituting $\lambda$ into (21) results in an expression for the optimal bit assignment $R_i$ to the scalar quantizer of subband $i$

\[
R_{i_{\text{opt}}} = \frac{1}{k} \log_2 \left( \frac{\alpha_i(p, r)}{\left[ \sum_{i=1}^{k} \alpha_i(p, r) \right]^{1/k}} \right).
\]

This expression can be used to evaluate the optimal bit allocation for the scalar quantizers if the pdf's of the subbands are known. By combining (23) with (7), (16a) and (18) finally
the desired distortion-rate function $D_s^R(R)$ is obtained

$$D_s^R(R) = \frac{2^{-r}}{r+1} \cdot \left( \sum_{i=1}^{r} \left[ \int [p(x_i)]^{(r+1)/2} \, dx_i \right]^{(r+1)/2} \right).$$

(24)

REFERENCES


Peter H. Westerink was born in The Hague, The Netherlands, on October 5, 1961. He received the M.Sc. degree in electrical engineering in 1985 from the Delft University of Technology, Delft, The Netherlands. Since 1985 he has been working towards his Ph.D. degree at the Delft University of Technology.

His interests include information theory, image coding, image restoration and digital signal processing.

Dick E. Boekee was born in The Hague, The Netherlands, in 1943. He received the M.Sc. and Ph.D. degrees in electrical engineering in 1970 and 1977, respectively, from the Delft University of Technology, Delft, The Netherlands.

In 1981 he became a Professor of Information Theory at the Delft University of Technology. During 1979–1980 he was a Visiting Professor at the Department of Mathematics, Katholieke Universiteit Leuven, Heverlee, Belgium. His research interests include information theory, image coding, cryptography, and signal processing.

Jan Biemond (M’80–SM’87) was born in De Kaag, The Netherlands, on March 27, 1947. He received the M.Sc. and Ph.D. degrees in electrical engineering from Delft University of Technology, Delft, The Netherlands, in 1973 and 1982, respectively.

He is currently an Associate Professor in the Laboratory for Information Theory of the Department of Electrical Engineering at Delft University of Technology. His research interests include multidimensional signal processing, image enhancement and restoration, data compression of images, and motion estimation with applications in image coding and computer vision. In 1983 he was a Visiting Researcher at Rensselaer Polytechnic Institute, Troy, NY, and at Georgia Institute of Technology, Atlanta, GA.

Dr. Biemond is a member of the IEEE-ASSP Technical Committee on Multidimensional Signal Processing. He has served as the General Chairman of the Fifth ASSP/EURASIP Workshop on Multidimensional Signal Processing, held at Noordwijkerhout, The Netherlands, in September 1987.


Since 1976 he has been with the ECSE Department at Rensselaer Polytechnic Institute, Troy, NY, where he is currently Professor. He has authored or coauthored over 40 papers on estimation, signal processing, and coding of images and other multidimensional data. He has coauthored one text in the area of probability, random processes, and estimation. During the academic year 1985–1986, he was Visiting Professor in the Information Theory Group at Delft University of Technology, the Netherlands. He is presently directing the Circuits and Signal Processing program at the National Science Foundation, Washington, DC.

Dr. Woods was cochair of the 1976 Best Paper Award of the IEEE Acoustics, Speech, and Signal Processing (ASSP) Society. He is a former Associate Editor for Signal Processing of the IEEE TRANSACTIONS ON ACOUSTICS, SPEECH, AND SIGNAL PROCESSING. He was cochairman of the Third ASSP Workshop on Multidimensional Signal Processing held at Lake Tahoe, CA, October 1983. He is a former Chairman of the ASSP Technical Committee on Multidimensional Signal Processing. He is currently an elected member of the ASSP Administrative Committee.